Two dimensional Nambu sigma model

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Abstract

We present two dimensional sigma model by using the Nambu structure on a manifold in general, and on a Lie group as a special case. Then, we consider model constructed from Nambu structure of order three and obtain conditions under which this model is equivalent to a WZW model. Furthermore, we present an example for this case on the four dimensional Heisenberg Lie group. Finally, as another example we show that the model constructed with Nambu structure of order three on the central extension of the 2D Poincare Lie group is integrable.

1 Introduction

Two dimensional sigma models have important role in string theories [1]-[3], statistical and quantum mechanical solvable models [4],[5]. Their symmetries are deeply depend on the geometry of their target spaces. The metric and antisymmetric tensor fields in the sigma model in general have no special structures. Of course up to now there are some attempts to use the special geometric structures in the model. For example, in the Poisson sigma model [6] the Poisson structure is used in the model. The advantage of this model is that it contains other two dimensional field theories (such as two dimensional Yang-Mill, two dimensional gravity, BF model, topological sigma model and gauged WZW models) as special cases. Another example is the Hitchin sigma model [7] such that in these models the generalized complex geometry conditions are obtained from a master equation of the master action (sigma model) [8] containing the generalized geometric structures. Our main idea is the construction of different sigma models from different geometric structures on the manifolds [9]. In this direction, in our previous work [10] we presented *Nijenhuis sigma model*. Here, we present *Nambu sigma model* constructed from Nambu structure on the manifold.

The paper is organized as follows. In section two we review some basic definitions and properties of the Nambu structure. Then, in section three by use of the Nambu structures of any order (even or odd) and an antisymmetric tensor of second rank, we present Nambu sigma models; specially we focus on the models constructed by Nambu structure of order three and investigate these models on Lie groups. After obtaining the conditions under which the Nambu sigma model is equivalent to the WZW model; in section four we present two examples: the Nambu sigma models on the Heisenberg and 2D extantion of Poincare Lie group. In the former we show that the model is equaivalent to the WZW models and then in section five we show that the second model is an integrable model.

2 Some basic definitions

In this section for self containing of the paper we review some basic definitions on Nambu structure. In 1973 Nambu [11] studied a dynamical system which was defined as a Hamiltonian system with respect to Poisson-like bracket, defined by a Jacobian determinant. About two decades later Takhtajan [12] by using an axiomatic formulation for n-bracket introduced the concept of Nambu structure and gave the basic properties and geometric formulations of Nambu manifolds. This new approach motivated a series of papers about some new concepts in relation to Nambu structure; also another generalization was the so-called generalized Poisson bracket [13]; (a

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comparison of both concepts was given in[14]). A C^∞ manifold endowed with a Nambu tensor (a skew-symmetric contravariant tensor field) is called Nambu manifold if the induced bracket satisfies the fundamental identity, which is generalization of the usual Jacobi identity [15]-[19]. Let M be a smooth n-dimensional manifold and $C^\infty(M)$ denotes the algebra of differentiable real-valued functions on M. A Nambu structure of order m is given by an m-dimensional multivector field, i.e. a $C^\infty(M)$-skew multilinear map

$$\eta : \Omega(M) \times ... \times \Omega(M) \longrightarrow C^\infty(M), \tag{1}$$

where $\Omega(M)$ is the set of one forms on M; such that in terms of the local coordinates $(x^1, x^2, ...x^n)$ we have

$$\eta = \eta^{\mu_{i_1}\cdots\mu_{i_m}}(x)\frac{\partial}{\partial x^{\mu_{i_1}}} \wedge \frac{\partial}{\partial x^{\mu_{i_2}}} ... \wedge \frac{\partial}{\partial x^{\mu_{i_m}}}, \tag{2}$$

where summation over repeated indices is understood. In this manner one can define the bracket of m functions $f_1, ..., f_m \in \mathcal{F}(\mathcal{M})$ as follows:

$$\{f_1, f_2, ..., f_m\} = \eta(df_1, df_2, ..., df_m), \tag{3}$$

note that the left hand side of the above equation is the Nambu bracket of m functions which is the generalize action of the Poisson bracket. Furthermore, since the bracket satisfies Leibnitz rule, one can define a vector field $X_{f_1,...,f_{n-1}}$by

$$X_{f_1,...,f_{n-1}}(g) = \{f_1, ..., f_{n-1}, g\} \quad \forall f_1, ..., f_{n-1}, g \in \mathcal{F}(\mathcal{M}), \tag{4}$$

where this vector field is called *Hamiltonian* vector field; the space of *Hamiltonian* vector field is denoted by $\mathcal{H}$.
Definition 1:[19]-[21]*An element $\eta \in \Gamma(\Lambda^m TM)$, for $m \geq 3$ is called a Nambu tensor of order m if it satisfies $\mathcal{L}_X \eta = 0$, for all $X \in \mathcal{H}$; where $\mathcal{L}$ stands for Lie derivative.*
The above definition is clearly equivalent to the following fundamental identity [12]

$$\begin{aligned}\{f_1, ..., f_{n-1}, \{g_1, ..., g_n\}\} = \{\{f_1, ..., f_{n-1}, g_1\}, g_2..., g_n\} + \{g_1, \{f_1, ..., f_{n-1}, g_2\}, g_3..., g_n\} \\ +... + \{g_1, ...g_{n-1}, \{f_1, ..., f_{n-1}, g_n\}\},\end{aligned} \tag{5}$$

for all $f_1, ..., f_{n-1}, g_1, ..., g_n \in C^\infty(M)$. If $n = 2$, this equation is nothing but the Jacobi identity for the Poisson structures. In Ref. [22], we have found the Nambu structures of order three and four on real four dimensional Lie groups. Now, in the next section by use of the Nambu structures of any order in general, and as an example in the especial case of order three we present a two dimensional sigma model (*Nambu sigma model*) on a manifold in general and on a Lie group as a special case. As an example, some sigma models constructed from Nambu structure of order $m = 3$ are also considered.

3 Nambu-Sigma model

Now, we assume that the manifold M has the Nambu structure up to top order n ($\eta^{\mu_1,\cdots,\mu_n}$), a 2-form $w_{\mu\nu}$ and also a 3-form $H_{\mu\nu\lambda}$; then one can construct the following actions by using of the Nambu structure with even order and odd order, respectively[1]

$$S_1 = \sum_{j,l=1}^{2k} \int_\Sigma d^2\sigma \varepsilon^{\alpha\beta} \eta^{\mu_{i_1}\mu_{i_2}\cdots\mu_{i_{2k}}} w_{\mu_{i_1}\mu_{i_2}} ... \hat{w}_{\mu_{i_j}\mu_{i_l}} ... w_{\mu_{i_{2k-1}}\mu_{i_{2k}}} G_{\mu_{i_j}\lambda} G_{\mu_{i_l}\nu} \partial_\alpha x^\lambda \partial_\beta x^\nu, \tag{6}$$

$$\begin{aligned}S_2 = \sum_{j,l=1}^{2k+1} \int_B d^3\sigma \varepsilon^{\alpha\beta\gamma} \eta^{\mu_{i_1}\mu_{i_2}\cdots\mu_{i_{2k+1}}} H_{\mu_{i_1}\mu_{i_2}\mu_{i_3}} ... \hat{H}_{\mu_{i_j}\mu_{i_l}\mu_{i_p}} ... H_{\mu_{i_{2k-1}}\mu_{i_{2k}}\mu_{i_{2k+1}}} \\ \times G_{\mu_{i_j}\rho} G_{\mu_{i_l}\nu} G_{\mu_{i_p}\lambda} \partial_\alpha x^\rho \partial_\beta x^\nu \partial_\gamma x^\lambda,\end{aligned} \tag{7}$$

[1]Here "ˆ" stands for omitted term.

where B is a three-dimensional manifold with boundary $\partial B = \Sigma$, and the coordinates x^μ are maps from Σ to M(such that those can be extend in an arbitrary fashion to the maps from B to M). In this way, by adding these actions to the following chiral action:

$$S_{ch} = \int_\Sigma d^2\sigma G_{\mu\nu}\partial_\alpha x^\mu \partial^\alpha x^\nu, \tag{8}$$

we construct new two dimensional sigma models, which we call two dimensional *Nambu sigma models.*[2] Note that these models in general are not conformal or integrable, and one can obtain conditions on η, w and H under which these models have the above properties.
Here, among these models we focus on the following model constructed only by Nambu structure of order three

$$S_{Ns} = \int_\Sigma d^2\sigma G_{\mu\nu}\partial_\alpha x^\mu \partial^\alpha x^\nu + k\int_B d^3\sigma \epsilon_{\alpha\beta\gamma}\eta^{\mu\nu\lambda}G_{\mu\eta}G_{\nu\rho}G_{\lambda\sigma}\partial^\alpha x^\eta \partial^\beta x^\rho \partial^\gamma x^\sigma. \tag{9}$$

Note that for this cases the forms w and H have disappeared and also k is a parameter. In this general form, this new model is not conformal or integrable; but one can find conditions by imposing conformality or integrability to the model. Here, we focus on the model (9) over a Lie group. The Nambu sigma model on a Lie group G can be written as follows:

$$S_{Ns} = \int_\Sigma \Omega_{ij}(g^{-1}dg)^i \wedge (g^{-1}dg)^j + k\int_B \eta^{ijk}\Omega_{il}\Omega_{jm}\Omega_{kn}(g^{-1}dg)^l \wedge (g^{-1}dg)^m \wedge (g^{-1}dg)^n, \tag{10}$$

$\forall g \in G$, such that with the assumption $\{X_i\}$ as a basis of Lie algebra $\mathbf{g}$ of the Lie group G we have $g^{-1}dg = (g^{-1}dg)^i X_i = e^i{}_\mu dx^\mu$. Here, $e^i{}_\mu$ is vierbein such that the metric and Nambu structure of order three on the Lie group G can be written as $G_{\mu\nu} = e^i{}_\mu \Omega_{ij} e^j{}_\nu$, $\eta^{\mu\nu\lambda} = \eta^{ijk} e_i{}^\mu e_j{}^\nu e^\lambda{}_k$ where Ω_{ij} and η^{ijk} are the metric and Nambu structure on the Lie algebra $\mathbf{g}$, and $e_i{}^\mu$ is the inverse of $e^i{}_\mu$. Note that when $\eta^{\mu\nu\lambda}$ is a constant or is a linear function of the coordinates x^μ, then our model ((9) or (10)) in general is not conformal or integrable. Here we obtain condition on the algebraic Nambu structure η^{ijk} such that the second term in the model (10) is the same as the WZW term. We know that the WZW term can be written as follows:

$$I_{wzw} = \int_B < (g^{-1}dg) \wedge [g^{-1}dg \wedge g^{-1}dg] >= \int_B (g^{-1}dg)^l \wedge (g^{-1}dg)^m \wedge (g^{-1}dg)^n f^i{}_{mn}\Omega_{li}. \tag{11}$$

Now by comparing this term with the second term of (10) we will obtain the following relation between the Nambu structure η^{ijk} and structure constant $f^i{}_{jk}$ on the Lie algebra

$$f^i_{mn} = \eta^{ijk}\Omega_{jm}\Omega_{kn}, \tag{12}$$

or in the matrix form

$$\mathcal{Y}^i = -\Omega\eta^i\Omega, \tag{13}$$

where $(\mathcal{Y}^i)_{mn} = -f^i_{mn}$ is the adjoint representation of the Lie algebra $\mathbf{g}$ and $(\eta^i)^{kj} = \eta^{ikj}$. Note that this relation is compatible with the ad-invariant metric Ω (the metric satisfies the relation $\chi_i\Omega = -(\chi_i\Omega)^t$ with $(\chi_i)_m^n = -f^n_{im}$). In this way, we see that for some special examples, where (13) holds, the Nambu sigma model is reduced to the WZW model as a special case. In other words according to the C theorem [24] this Nambu sigma model with the above property is a fixed point of the RG flow of the general Nambu sigma model with Nambu structure of order three. Now, in the following we present an example for this case and also other example for the model (10) on the Lie group which is integrable.

4 Examples

In this section we consider two examples for the Nambu sigma model on a Lie group with Nambu structure of order three (model (10)).

[2]Note that these sigma models, are different from the models presented in [23].

a) We first consider the four dimensional Heisenberg Lie group. This Lie group has Lie algebra which is isomorphic with the $A_{4,8}$ in the Petra and *et al.* [25] classification of four dimensional real Lie algebras. We have the following commutation relations for this algebra

$$[P_2, T] = P_2, \quad [P_2, J] = P_1, \quad [T, J] = J, \tag{14}$$

where we use the generators $X_i = \{P_1, P_2, J, T\}$. Recently in [22], we have obtained the Nambu structures of order four and three for its related Lie sub group; such that a Nambu structure of order three has the following form

$$\eta = (q_2 a_2 + q_3 u)\frac{\partial}{\partial a_1} \wedge \frac{\partial}{\partial a_2} \wedge \frac{\partial}{\partial u}. \tag{15}$$

By use of the following parametrization for the Lie group elements $g \in A_{4,8}$

$$g = e^{\Sigma_i a_i P_i} e^{uJ} e^{vT}, \tag{16}$$

and vierbein [22]

$$e^i{}_\mu = k \begin{pmatrix} 1 & u & 0 & 0 \\ 0 & e^v & 0 & 0 \\ 0 & 0 & e^{-v} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{17}$$

and after some calculations we obtain

$$\eta^{ijk} = -1. \tag{18}$$

We see that for this example the relation ((12) or (13)) holds with the following ad-invariant metric for $A_{4,8}$ [22]

$$\Omega_{ij} = \begin{pmatrix} 0 & 0 & 0 & -k \\ 0 & 0 & k & 0 \\ 0 & k & 0 & 0 \\ -k & 0 & 0 & b \end{pmatrix}. \tag{19}$$

In other words, the Nambu model (10) on Lie group $A_{4,8}$ is reduced to the WZW model.

b)The second model we will consider in the following is the Nambu sigma model based on a certain non-semi-simple Lie algebra of dimension four. This Lie algebra has the following explicit description:

$$[J, P_i] = \varepsilon_{ij} P_i, \quad [P_i, P_j] = \varepsilon_{ij} T, \quad [T, J] = [T, P_a] = 0. \tag{20}$$

Indeed this Lie algebra is a central extension of the 2D Poincaré algebra which is reduced to Euclidian algebra, when one sets $T = 0$. The corresponding simply-connected Lie group is called R [26][3].
For writing the action of the Nambu sigma model (10) we use the following parametrization of the Lie group manifold of the above Lie algebra [26]

$$g = e^{\Sigma_i a_i P_i} e^{uJ + vT}. \tag{21}$$

In this way, the left invariant 1-forms and vector fields are obtained as follows[26]

$$g^{-1}dg = (cosu\; da_k + \varepsilon_{jk} sinu\; da_k)P_k + d_k u J + (dv + \frac{1}{2}\varepsilon_{jk} a_k da_j)T, \tag{22}$$

$$X_1 = cosu\frac{\partial}{\partial a_1} + sinu\frac{\partial}{\partial a_2} - a_2 cosu\frac{\partial}{\partial v}, \tag{23}$$

$$X_2 = -sinu\frac{\partial}{\partial a_1} + cosu\frac{\partial}{\partial a_2} + a_2 sinu\frac{\partial}{\partial v}, \tag{24}$$

$$X_3 = \frac{\partial}{\partial u} \;\; X_4 = \frac{\partial}{\partial v}. \tag{25}$$

[3]Note that the Lie algebra (20) is isomorphic with the Lie algebra $A_{4,10}$ of [25].

In [22], we have obtained the Nambu structure of order four and three for this Lie group; where the Nambu structure of order three has the following form

$$\eta = (q_1 a_1 + q_2 a_2)\frac{\partial}{\partial a_1} \wedge \frac{\partial}{\partial a_2} \wedge \frac{\partial}{\partial v}. \tag{26}$$

The Nambu structure of order three on the extension of the 2D Poincare can also be written as follows [22]:[4]

$$J = \frac{\partial}{\partial a_1} \wedge \frac{\partial}{\partial a_2} \wedge \frac{\partial}{\partial v}, \tag{27}$$

such that $\eta^{124} = 1$. Now using (22) and the following Lie algebra metric Ω_{ij} [26]

$$\Omega_{ij} = k \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & b & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{28}$$

one can obtain the first term of the action (10) as the first term of (9) with the following metric [26]

$$G_{\eta\mu} = \begin{pmatrix} 1 & 0 & \frac{a_2}{2} & 0 \\ 0 & 1 & \frac{-a_1}{2} & 0 \\ \frac{a_2}{2} & \frac{-a_1}{2} & b & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{29}$$

In the same way, using the above Nambu structure (27) and the Lie algebra metric Ω_{ij}, the second term of Nambu sigma model action (10) can be integrated as follows:

$$\int_\Sigma d^2\sigma 2\varepsilon_{\alpha\beta}(a_1\partial^\alpha a_2 \partial^\beta u + u\partial^\alpha a_1 \partial^\beta a_2 + a_2 \partial^\alpha u \partial^\beta a_1), \tag{30}$$

such that by comparison of the above term with the second term of the following action

$$S_{Ns} = \int_\Sigma d^2\sigma (G_{\mu\nu}\partial_\alpha x^\mu \partial^\alpha x^\nu + B_{\mu\nu}\varepsilon_{\alpha\beta}\partial^\alpha x^\mu \partial^\beta x^\nu), \tag{31}$$

we have

$$B_{\mu\nu} = \begin{pmatrix} 0 & u & -a_2 & 0 \\ -u & 0 & a_1 & 0 \\ a_2 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{32}$$

where in this action $x^\mu = (a_1, a_2, u, v)$. By comparing this action with the WZW action on R [26], i.e.,

$$S_{wzw}(g) = \int_\Sigma d^2\sigma (G_{\mu\nu}\partial_\alpha X^\mu \partial^\alpha X^\nu + iB_{\mu\nu}\varepsilon_{\alpha\beta}\partial^\alpha X^\mu \partial^\beta X^\nu), \tag{33}$$

we find out that the only difference between two models, is the antisymmetric matrix B, whose non zero components in WZW action is just $B_{12} = u$, whereas in Nambu sigma model we have other non zero components $B_{13} = -a_2,\ B_{23} = a_1$; such that this model is not conformal. Here, we prove that this model is integrable. For this purpose, in the following section we use the method presented by Mohammedi [27].

5 The integrability of the Nambu-Sigma model on the central extension of the 2D Poincare Lie group

In this section for self containing of the paper we give a short review on the integrability of a sigma model, recently presented by Mohammedi in [27]. Consider the following Sigma model action

$$S_{Ns} = \int_\Sigma d^2\sigma (G_{\mu\nu}\delta^\beta_\alpha + B_{\mu\nu}\varepsilon^\beta_\alpha)\partial^\alpha x^\mu \partial_\beta x^\nu, \tag{34}$$

[4] Indeed this is the Nambu structure of top order for the Lie subalgebra of 2D Poincare algebra with basis $A_{3,1} : \{X_1, X_2, X_4\}$[22].

the equations of motion for this model can be written as the following Lax pair

$$[\partial_0 + \alpha_\mu(x)\partial_0 x^\mu]\psi = 0, \tag{35}$$

$$[\partial_1 + \beta_\nu(x)\partial_1 x^\nu]\psi = 0, \tag{36}$$

if the matrices $\alpha_\mu(x)$ and $\beta_\nu(x)$ satisfy the following relations [27]

$$\beta_\mu - \alpha_\mu = \mu_\mu, \tag{37}$$
$$\partial_\mu\beta_\nu - \partial_\nu\alpha_\mu + [\alpha_\mu, \beta_\nu] = \Omega^\lambda_{\mu\nu}\mu_\mu, \tag{38}$$

with

$$\Omega^\tau_{\mu\nu} = \Gamma^\tau_{\mu\nu} - H^\tau_{\mu\nu}, \tag{39}$$

where equation (38) can then be rewritten by splitting symmetric and anti-symmetric parts as follows:

$$0 = \nabla_\mu\mu_\nu + \nabla_\nu\mu_\mu - 2\Gamma^\tau{}_{\mu\nu}, \tag{40}$$
$$F_{\mu\nu} = -\frac{1}{2}(\nabla_\mu\mu_\nu - \nabla_\nu\mu_\mu) - H^\tau{}_{\mu\nu}\mu_\tau. \tag{41}$$

In the above formula $\Gamma^\tau_{\mu\nu}$ is the chiristofel coefficients and

$$H^\tau_{\mu\nu} = \frac{1}{2}g^{\tau\lambda}(\partial_\lambda b_{\mu\nu} + \partial_\nu b_{\lambda\mu} + \partial\mu b_{\nu\lambda}), \tag{42}$$

such that the field $F_{\mu\nu}$ and covariant derivative corresponding to the matrices α_μ are given as follows:

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu + [\alpha_\mu, \alpha_\nu], \\ \nabla_\mu X &= \partial_\mu X + [\alpha_\mu, X]. \end{aligned} \tag{43}$$

In this manner, the integrability condition of the sigma model (34) is equivalent to finding the matrices α_μ, μ_μ such that they satisfy (40) and (41).
Now, we apply this formalism to the two dimensional Nambu sigma model (31) on the non-semi-simple Lie algebra (20). In order to find some solutions, we proceed by fixing some of these unknowns quantities with the assumption

$$\begin{aligned} \alpha_\mu &= x e^i_\mu X_i, \\ \mu_\mu &= y e^j_\mu X_j, \end{aligned} \tag{44}$$

where the indices of the Lie algebra $i, j, k, ...$ have the same range as those of the target space of the sigma model $\mu, \nu, ...$ We will use the fact that the gauge condition $A_\mu = g^{-1}\partial_\mu g = e^i{}_\mu T_i$ satisfies the Bianchi identity $\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] = 0$. As mentioned above, the inverse of the vierbains $e^i{}_\mu$ are denoted by $e_i{}^\mu$ such that we have $e^i{}_\mu e_j{}^\mu = \delta^i_j$ and $e^i{}_\mu e_i{}^\nu = \delta^\nu_\mu$. We assume that the quantities x and y are two constant parameters different from zero. Inserting the expressions of α and μ in equations (40) and (41) leads to

$$\Gamma^\tau_{\mu\nu} = \frac{1}{2}e_i{}^\tau(\partial_\mu e^i{}_\nu + \partial_\nu e^i{}_\mu), \tag{45}$$
$$H^\tau_{\mu\nu} = \kappa e^i{}_\mu e^j{}_\nu e_k{}^\tau f^k_{ij}\ , \tag{46}$$

with $\kappa = \frac{-1}{y}(x^2 - x + xy - \frac{1}{2}y)$.
The above Chiristoffel symbols $\Gamma^\tau_{\mu\nu}$ and torsion $H^\tau_{\mu\nu}$ are those corresponding to the following metric $g_{\mu\nu}$ and anti-symmetric tensor $b_{\mu\nu}$

$$g_{\mu\nu} = \Omega_{ij}e^i{}_\mu e_\nu{}^j, \tag{47}$$
$$H_{\mu\nu\tau} = \kappa\Omega_{li}f^l_{jk}e^j{}_\mu e^k{}_\nu e_\tau{}^i, \tag{48}$$

where Ω_{ij} is an invertible bilinear form of the corresponding Lie algebra satisfying $\Omega_{ij} f^j_{kl} + \Omega_{kj} f^j_{il} = 0$. By solving the equations (45) and (46) we find that $\kappa = \frac{3}{2}$ for the Nambu sigma model. To summarise, the Lax pair construction for the Nambu sigma model represented by the metric and torsion in (47) and (48) is given by

$$[\partial + x(g^{-1}\partial_\mu g)\partial\varphi^\mu]\psi = 0, \tag{49}$$

$$[\bar{\partial} + \frac{7x}{2x+2}(g^{-1}\partial_\nu g)\bar{\partial}\varphi^\nu]\psi = 0. \tag{50}$$

6 Conclusion

Here, we have presented two dimensional Nambu sigma models on a manifold in general and on a Lie group as a special case. In general, these models are not conformal or integrable and one can obtain conditions under which they have these properties. When the Nambu structure on a Lie group is of order three, we have obtained conditions under which this model is equivalent to a WZW model. Moreover, we have presented an example on the Heisenberg Lie group for this case. As another example, we have presented the model constructed by the Nambu structure of order three on the central extension of the 2D Poincare Lie group, such that this model is integrable.

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